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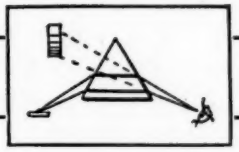

APRIL
1934

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Highly Reflecting Films of Zinc Sulphide

A. H. PFUND, *The Johns Hopkins University*

(Received January 8, 1934)

The initially high reflection from zinc sulphide is still further enhanced by constructive interference in thin films. By distilling sphalerite in vacuum, thin films showing first order interference and reflecting white light brilliantly are produced. The application of such films to the metallographic microscope, Michelson interferometer and other instruments is discussed.

IF a wedge-shaped film of zinc sulphide be formed, a series of interference colors similar to those exhibited by a vertical soap-film will be observed. In both cases, the order of colors, away from the thin edge of the wedge is: black, bluish-grey, white, yellow, blue-red, blue, etc. While it is not necessary to repeat the long-known explanation[1] of these colors, it is to be pointed out that, in the first interference maximum (Fig. 1) there is a region in which there is not much separation between the fringes for blue, green and red light. In this region the reflected light is white. In consequence of interference the amplitudes of rays 1 and 2 are added and hence it is possible to realize a film whose reflecting power for white light is much greater than that of a single polished surface of the same material.

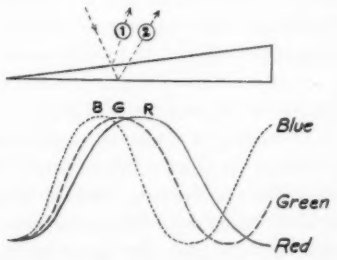

FIG. 1.

For the production of such "white" films, zinc sulphide (sphalerite) was chosen, first, because of its high refractive index ($n=2.37$) and, secondly, because of its volatility—in consequence of which this material may be distilled, with but little decomposition, from an incandescent tungsten spiral in high vacuum. The simple distillation apparatus previously described[2] was here again employed. To obtain white films over considerable areas, the microscope cover-glasses to be coated were mounted 3–4 cm below the tungsten spiral as well as along the sides of the bell jar. By repeated trials it was found that by reducing the pressure below 5×10^{-4} mm and by raising the tungsten spiral to the same white heat and by standardizing the size of the sphalerite fragment, brilliantly white films were deposited in 20 seconds.

[1] Müller-Pouillet, Vol. II, pp. 693, 1926.

[2] A. H. Pfund, J. O. S. A. 23, 375 (1933).

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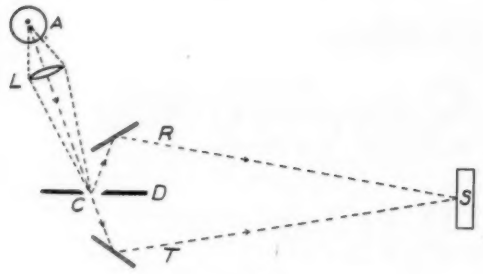

FIG. 2.

The transmission, reflection and absorption of these films was studied by means of the very convenient apparatus shown in Fig. 2. Here, *A* is an automobile headlight lamp whose filament image, formed by the lens *L*, is projected into the center of an opening *C* in the vertical plate *D*. Lamp and lens are mounted on an arm pivoted in line with *C* so as to make possible changes in the angle of incidence. The film-coated plate which is to be tested (not shown) is mounted over this opening so that the reflected beam *R* and the transmitted beam *T* are finally thrown on the oversize surface of a Weston photronic cell *S* (having no central button). The cell is simply connected to a sensitive millivoltmeter whose readings are assumed to be directly proportional to light intensity. The mode of operation of the apparatus is very simple: when the opening *C* is free, the total incident light, travelling along path *T*, is measured; upon introducing the cover-glass coated with zinc-sulphide, the transmitted and reflected intensities may be measured, either separately, or together. Obviously, the difference between the incident intensity and the sum of the reflected and transmitted intensities is equal to the light absorbed within the film.

The results given in Table I were all obtained for an angle of incidence of 45°; when calculations were necessary the formula of Fresnel was employed to determine the reflection coefficient when the incident unpolarized light was allowed to fall on the reflecting surface under 45° incidence. The refractive indices of glass and zinc-sulphide were taken as 1.5 and 2.37, respectively.

TABLE I.

Reflector	Reflecting power
Single glass surface	5% calculated
" ZnS "	17.3 "
Clear cover glass (2 surfaces)	10.0 observed
Single white ZnS film on glass	31. "
White ZnS film on both sides of cover glass	40. "

From these results the superiority of the zinc sulphide films is evident. That their full theoretical performance is not realized may be attributed largely to the circumstance that one side of the film is in contact with glass. Simultaneous transmission and reflection measurements showed that even the cover-glass with ZnS films on both sides absorbed less than 3 percent of the total incident energy. Similar measurements on an excellent "half-silvered" mirror, deposited chemically, gave the following results: transmission 50 percent, reflection 27 percent, absorption 23 percent.

The principal interest in white zinc-sulphide films lies in their application to mirrors which first reflect the light and, later, transmit it. Such mirrors are called for in the construction of a Gaussian eyepiece, Michelson interferometer, metallographic microscope, etc. Of these, the last named is probably the most important. It is well known that a mirror reflecting 50 percent and transmitting 50 percent will yield the theoretical maximum of 25 percent upon reflection-transmission. Metallic films fall short of this limit because of the relatively large absorption in the metal layer. In the preparation of a zinc sulphide reflector a large cover-glass 43×70×0.2 mm, coated with a "white" film on both sides, was used. Such a reflector transmits 57 percent, reflects 41 percent and absorbs 2 percent—the resultant reflection—transmission being 23 percent.

The actual performance of such a reflector in a metallographic microscope is shown in the photomicrographs (Fig. 3) kindly prepared by Dr. J. B. Clough of the Brandywine Laboratory of the Du Pont Company. The same etched specimen of wrought iron was photographed (160 diameters) in green light, first, with the usual clear glass reflector, and subsequently, with the ZnS reflector. A comparison of the photographs shows that the light intensity delivered by the ZnS reflector is clearly greater than twice that given by the clear glass. While the performance of these new reflectors falls but

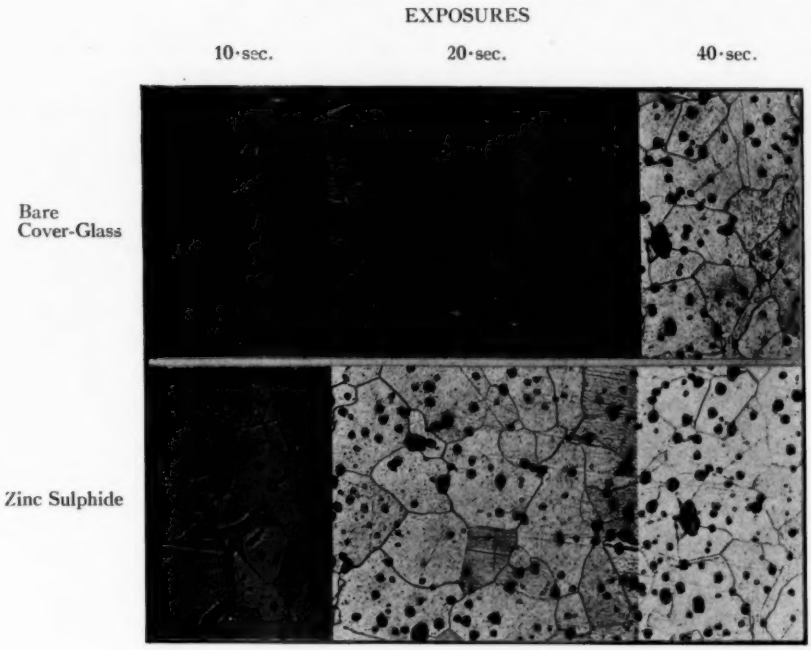

FIG. 3.

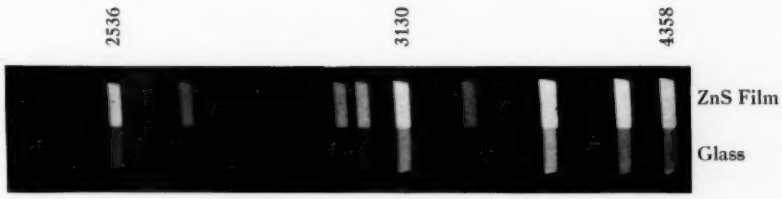

FIG. 4. Relative intensities of reflection from ZnS and glass.

little short of the theoretical maximum, the zinc-sulphide films are found to be soft and rather easily scratched. Instead of attempting to clean a reflector, it is best to replace it by a fresh one.

A study of the performance of the Michelson interferometer has shown that upon coating one side of the first diagonal plate with a white ZnS film, the intensity of the fringes is greatly enhanced—as might have been anticipated. In spite of the use of the ZnS film, the phase-relations between the two interfering beams are such as to leave the usual "black" central fringe of the Michelson interferometer unaltered.

An attempt was made to determine, at least qualitatively, the reflecting properties of ZnS films in the ultraviolet. Since a decrease in the wave-length of light brings about a shift of the first interference maximum toward the thin edge of the wedge (Fig. 1), the former white films would have been too thick for the ultraviolet. Therefore a much thinner film, showing a bluish-grey color was prepared for this work. The film was removed from a portion of the glass plate and this was so mounted in front of a quartz spectrograph that the upper and lower halves of the slit were illuminated by mercury radiations reflected from ZnS and glass, respectively (Fig.

4). Conclusions based on the use of the reciprocity law showed that even down to 2300A the intensity of light reflected from the ZnS film is the greater in the ratio of 6 : 1—which is the same ratio as that previously found for a white film as against glass in visible light. No direct measurements extending into the Schumann region are at hand. Earlier studies carried out in 1927 showed that in the interval 2000–900A sphalerite showed no pronounced reflection maxima (as in the case of NaCl)[3] but rather, a high and constant reflecting power. Anticipating then, that the advantages gained through the use of a ZnS film up to 2300A are retained even in the region of the far ultraviolet, it is suggested that glass gratings for use at normal incidence be coated with a "first order" film. It may be remarked that zinc sulphide films are readily dissolved in dilute hydrochloric acid.

As stated in the beginning, ZnS films were used in this work because of the ease of deposition. It is more than probable, however, that even better results would be obtained with films of TiO_2 ($n=2.73$), stibinite ($n=4.3$) or other materials having very high refractive indices.

[3] A. H. Pfund, Phys. Rev. **32**, 39 (1928).

Zone Plate Determination of Dominant Wave-Length of Filtered Lamp Light

JOSEPH W. ELLIS AND BARTHOLD W. SORGE, *University of California at Los Angeles*

(Received January 30, 1934)

Use is made of the enormous inverse chromatic aberration of a zone plate to determine the dominant wave-length, so far as the eye is concerned, of the light from a lamp of definite color temperature transmitted through members of a group of gelatin filters. For representative members of the group this wave-length is compared with the center of gravity of the luminosity curve plotted with the aid of the energy distribution of the lamp, the transmission percentages for the filters and the visibility curve for the normal eye.

IN the physical optics laboratory it occasionally becomes desirable or necessary to use filtered white light in place of monochromatic light when making visual observations in connection with such experiments as straight-edge and single slit diffraction, double slit, Lloyd mirror and Fresnel mirrors interference, etc. Fairly standardized filters are now available in the Wratten gelatin filters supplied by the Eastman Kodak Company, but so far as the authors are aware no results of a study of dominant wave-length have been recorded. It is the purpose of this paper to present such results obtained by means of a zone plate for a large group of these filters; and particularly to describe the method of observation, inasmuch as the method may suggest other applications.

The dominant wave-length as determined in this experiment should be expected to agree fairly closely with the "center of gravity" of the luminosity curve plotted for a given filter. It is not the "wave-length of dominant hue," or "dominant hue," used by workers in colorimetry, which is defined as the wave-length of monochromatic light which must be mixed with white light to reproduce the given color.

The method of observation involves the use of a zone plate. That a zone plate made up of alternate transparent and opaque rings whose diameters vary as the square roots of successive integer numbers has the focussing properties of a lens was first proved by Soret,[1] who made so-called "Kreisgitters" both by ruling on glass plates and by photographically reducing larger drawings. It is interesting to note that Soret was able to get good images with filtered sunlight as well as with monochromatic light. This is true largely because of the relative sharpness of the visibility curve of the human eye.

The zone plate, regarded as a multiple focus lens, has an enormous inverted chromatic aberration. In fact, in any one order the focal length is inversely proportional to the wave-length of light. It is this dependence of focal length upon wave-length of which use is made in the present investigation. The equation which is obeyed is

$$1/f_\lambda = 1/u + 1/v_\lambda = nk\lambda/r_n^2, \qquad (1)$$

in which r_n is the radius of the nth ring and k is the order of the focus with values of 1, 2, 3, ···. Solving for the wave-length,

$$\lambda = r_n^2/nku + r_n^2/nkv_\lambda = A + B/v_\lambda, \qquad (2)$$

provided the u distance is kept constant. The determination of λ then is accomplished by finding the values of A and B and making observations on v_λ.

The zone plate used was a 63-fold photographic reduction on a contrast lantern slide plate of a drawing made years ago in this laboratory. The drawing had 175 zones and the diameter of the last zone was 58.4 cm. The reduced plate had a first order focal length of 24.29 cm for λ5461A. In the final experimental arrangement it was used with a Ramsden eyepiece to form a telescope with a magnification of approximately 9 diameters for this wave-length. The object

[1] J. L. Soret, Ann. d. Physik und Chemie **156**, 99 (1875).

viewed through this telescope was a small rectangular aperture in front of which was placed the filter whose dominant wave-length was to be measured and behind which was placed a 500-watt tungsten lamp of color temperature 2600°K. A steel scale was securely clamped parallel to the telescope and the v_λ values were read by means of a pointer fastened on the eyepiece tube in the plane of the cross-hairs.

Sixty-five Wratten filters supplied by the Eastman Kodak Company and representing a majority of the gelatin filters listed by this firm were used. Several of the lightest filters and most of the purple ones, which are transparent in two regions of the spectrum, were discarded. Each of two observers made ten determinations of v_λ for each of these filters, using a Huyghens eyepiece. Only first order values were recorded, although five orders were observable. When the band of color transmitted by a filter was narrow the observer adjusted the telescope for the maximum sharpness and brightness of image. When the band was broad he kept in mind not only maximum sharpness and brightness but he also adjusted for the position in which the halo surrounding the image was least noticeable. If, for example, an orange filter was under observation, as the eyepiece was pulled out and the critical value of v_λ was approached and exceeded, this halo changed from green to red.

After this set of observations had been completed it was apparent that the use of a Huyghens eyepiece made it impossible to secure acceptable determinations of v_λ. Accurate corrections for the refraction introduced by the field lens was not possible. A Ramsden eyepiece was then substituted for the previous one, and new observations were made upon a selected group of twenty filters, representing all the characteristic types and including those most likely to be found in a physical laboratory.

Throughout the investigation the value of u was kept at 400.0 cm. The values of A and B in Eq. (2) were obtained both from the slope of a $r_n^2 : n$ curve plotted from microscope determinations of r_n and by making a direct determination of v_λ for λ5461 monochromatic light from a mercury arc. Although these two determinations yielded values of r_n^2/n differing by only 0.4 percent, only the value determined by the latter method was used. That the zone plate was essentially plane was proved by finding no variation in f_λ upon reversing the plate.

The values of the dominant wave-lengths for the twenty selected filters as determined independently by observer A and observer B with

TABLE I.

Filter number	Dominant λ Obs. A	Percent m.d. Obs. A	Dominant λ Obs. B	Percent m.d. Obs. B	Dominant λ Av. $A+B$	C. of G.	λ(max)	Dominant hue
12	577 mμ	0.27	579 mμ	0.32	578 mμ	576 mμ	573 mμ	571 mμ
16	589	.13	588	.24	589	583	574	580
22	607	.18	602	.23	604	598	580	593
26	617	.26	621	.26	619	618	610	617
29	628	.12	625	.59	627	634	630	628
31*	618	.39	621	.26	619	630	630	—
35*	668	.39	668	.83	668	672	670	—
38A	521	.69	538	.43	530	541	541	485
47A	474	.75	472	.68	473	478	483	464
49A	474	.50	472	.88	473	484	486	466
49B	465	.98	468	.72	466	466	468	—
50	462	.77	457	.74	460	461	462	—
54	544	.74	546	.54	545	550	550	546
55	538	.48	542	.87	540	540	540	539
58	540	.39	546	.67	543	535	518	539
62	534	.40	533	.26	533	534	533	—
64	536	.68	545	.89	540	537	537	537
70	670	.36	678	.41	674	677	670	690
71A	623	.39	623	.49	623	629	627	639
74	530	.38	534	.84	532	533	532	535
Unfiltered	572	.38	577	.40	574	572	574	—

* These are purple filters, and the dominant wave-length given is for the red transmitted band only.

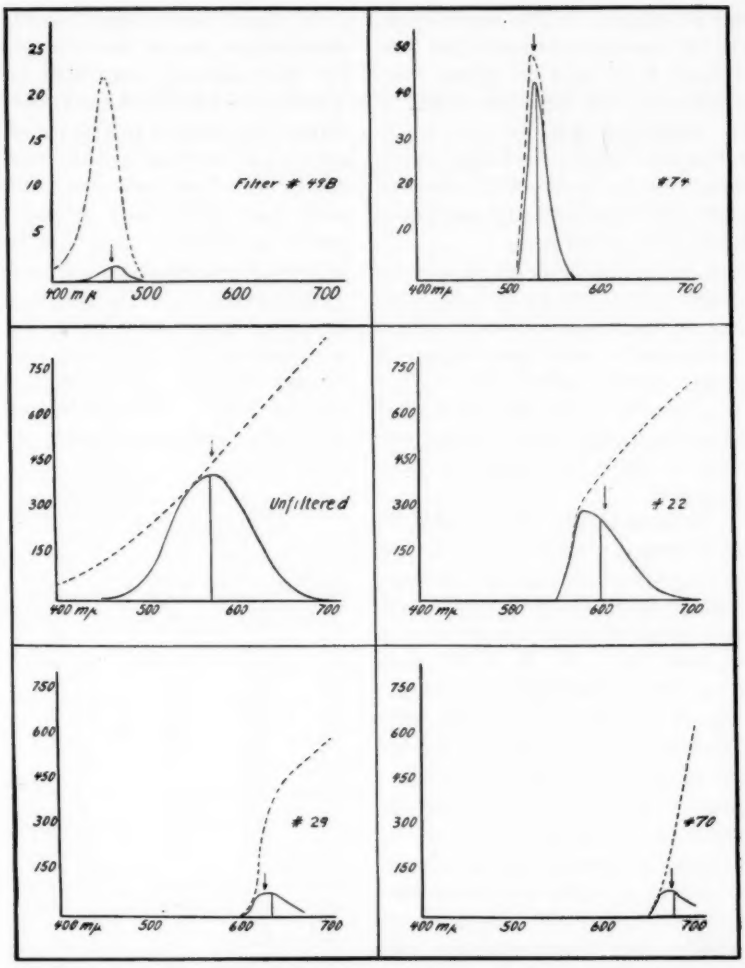

FIG. 1. Energy transmission (broken lines) and luminosity (full lines) curves for five selected filters and for unfiltered lamp light. Centers of gravity of luminosity curves are indicated by vertical lines. Zone plate determinations are indicated by arrows.

the percent mean deviation for each and the average value of both are given in columns 2–6 of Table I.

To obtain comparison data a luminosity curve for each of the twenty filters was drawn from data secured from the percent transmission data supplied with the filters, the accepted visibility data for normal observers adopted by the Optical Society of America[2] and the radiant energy of the lamp used with the filters. The ordinates on the luminosity curves were obtained from a product of these three factors for any one wave-length. The observers subjected themselves to the Holmgren and Stillings color blindness tests to justify their use of normal visibility data. The energy distribution of the lamp was determined with a Hilger quartz monochromatic illuminator equipped with a Hilger linear thermopile and connected with a high-sensitivity galvanometer. Corrections were made for the variation in dispersion of the quartz prism. The "center of gravity" of each luminosity curve has been determined and the values have

[2] P. G. Nutting, J. O. S. A. **4**, 55 (1920).

been tabulated in column 7. The wave-length associated with the maximum ordinate has been tabulated in column 8. In the final column have been inserted the values of the wave-length of dominant hue, kindly supplied by Dr. L. A. Jones of the Eastman Research Laboratories. These latter values are the results of the observations of a single individual and the maximum uncertainty is placed at about 6 mμ.

The dominant wave-lengths given in column 6 are not for "white" light, but are for light from a lamp of a definite color temperature,[3] 2600°K. This is the approximate color temperature of gas-filled tungsten lamps, which are readily accessible in any laboratory. If the filters were used with a lamp of any other color temperature the values given in the table would be only approximately correct.

The data for unfiltered light are included in the table. The observers were surprised to see the consistency with which they set the eyepiece for this light and the agreement of the dominant wave-length with the maximum of the luminosity curve for the lamp light. W. B. Croft[4] has briefly pointed out the complementary relationship between the color of the image formed with white light and its surrounding halo for any v_λ setting.[5] A study of the formation of the successive images shows that these colors cannot be strictly complementary. As the value of v_λ associated with the maximum luminosity is approached for unfiltered light, the image becomes brightest and reaches the lightest tint, while the surrounding halo becomes darkest and least conspicuous. This phenomenon is a feature of complementarity.

In Fig. 1 have been assembled the energy distribution curves and the luminosity curves for the unfiltered lamp light and for the light filtered through five of the twenty representative filters. The centers of gravity of the luminosity curves and the results of the zone plate measurements have been indicated. Although arbitrary units have been used in the plotting of the curves, a constant ratio has been maintained between the luminosity and energy ordinates.

Determination of the dominant wave-lengths for the remaining forty-five filters was finally accomplished by calibrating the original set-up through the use of the data on the twenty selected filters. These dominant wave-lengths have been tabulated in Table II. No luminosity curves have been drawn for these filters and consequently no centers of gravity or λ(max) values appear in this table. The wave-lengths of dominant hue supplied by Dr. Jones have been included however.

TABLE II.

Filter number	*Dominant wave-length	Dominant hue	Filter number	*Dominant wave-length	Dominant hue
5*N*5	578 mμ	—	48	479 mμ	—
11	562	—	48*A*	474	463 mμ
13	552	571 mμ	49	467	458
15	586	574	51	572	557
17	576	564	52	557	546
21	593	586	53	553	548
23	611	600	56	561	548
23*A*	614	597	57	541	541
23*B*	606	—	58*A*	541	541
24	619	607	59	542	534
24*A*	620	606	59*A*	558	538
25	621	610	60	536	524
27	619	607	61	539	534
27*A*	612	—	63	528	514
28	619	604	65	523	499
40	540	522	65*A*	519	—
40*A*	555	541	66	559	536
41	493	—	67	532	495
43	530	482	68	511	488
44*A*	518	488	69	527	507
45	504	482	72	591	618
45*A*	500	—	73	566	584
46	496	475			

* Determined from the average of twenty observations by two observers.

[3] I. G. Priest, J. O. S. A. **6**, 27 (1922).

[4] W. B. Croft, Nature **89**, 581 (1912).

[5] The literature of the zone plate appears to be very meager. In addition to the original contribution of Soret and this note by Croft, there are the discussion by R. W. Wood in his *Physical Optics* and his publication (Phil. Mag. **45**, 511 (1898)) of his successful construction of phase reversal zone plates, the possibility of which had been predicted by Lord Rayleigh in his article on *Wave Theory* in the early editions of the *Encyclopaedia Brittanica*.

The Dependence of Flicker on the Dark-Light Ratio of the Stimulus Cycle

PERCY W. COBB, *Department of Ophthalmology, Washington University*
(Received February 1, 1934)

IN an earlier communication[1] the theory of flicker proposed by Ives[2] was discussed, with reference to the data presented by him. These results had to do with an abruptly changing stimulus with equal light and dark intervals, and with a stimulus of simple sinusoidal time pattern, both with varying amplitudes of fluctuation.

The present paper has experimental data to contribute to another phase of the question. That is, the relation between the ratio φ of duration of light phase to the whole period, and the critical speed. Ives' data on this aspect of the question (p. 262, Fig. 4, right-hand section)[2] are here reproduced for comparison in Fig. 1.

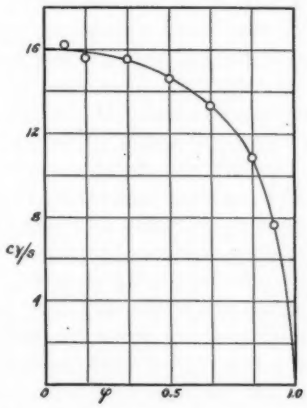

FIG. 1. Critical frequency, in cycles per second, plotted against φ, the ratio of the light phase to the period. Ives' data, using dim spectral blue light, within a range in which critical frequency does not change with intensity.

These were obtained by using a stimulus which was abruptly interrupted and abruptly resumed, consisting of spectral blue light within a range of low intensity in which it had been shown that the critical speed did not vary with intensity. In general, as the open phase of the stimulus becomes a larger fraction of the cycle time the average intensity admitted to the eye increases in proportion, and the effect on the visual apparatus increases accordingly, following the Talbot-Plateau law. Under ordinary circumstances we could then expect an increase in critical speed from this intensity factor alone, and would have no way of separating this from the effect sought, which is that due to the change in the time pattern alone. Ives has avoided this difficulty, as indicated, by selecting conditions under which the intensity effect was demonstrably absent, that is, by using spectral blue light within a certain range of low intensity.

In the earlier paper[1] I proposed a different method of accomplishing the same result, which will best be made clear by an example.

Suppose that we wish to know the effect of the stimulus pattern alone on critical frequency when a cycle in which the light period is one-half as long as the dark ($\varphi=\frac{1}{3}$) is changed to one in which the light period is twice the length of the dark ($\varphi=\frac{2}{3}$). The first pattern admits to the eye one-third of the light incident on the disk, the second two-thirds. If, then, the light intensity before interruption is made twice as great in the first case as in the second, the average light admitted to the eye will be equal in the two cases; and according to the Talbot-Plateau law, at higher-than-critical speeds the two will be identical in their effect on the visual apparatus. It is fair to assume that this identity of effect will extend further than the mere appearances of intensity and color, and will cover also the effect of the light upon the adaptive state of the eye, that is, upon its known adjustment to intensity, and that this adjustment will not be materially altered when the speed is reduced to the point at which flicker is no more than clearly observ-

[1] P. W. Cobb, *Some Comments on the Ives Theory of Flicker*, J. Opt. Soc. Am. **24**, 91–98 (1934).

[2] H. E. Ives, (a) *Critical Frequency Relations in Scotopic Vision*, J. Opt. Soc. Am. **6**, 254–268; (b) *A Theory of Intermittent Vision*, 343–361 (1924).

able. And that is all the variation in speed that is practically required in that direction, in order to make a setting at the critical frequency.

The results to be presented were obtained by this method; the intensity of the original uninterrupted field was always varied in inverse proportion to the ratio of light period to cycle time, so that the average light intensity for the eye was always equal to a selected standard for any particular series. The adaptation of the visual apparatus was thus equalized, and the variations in critical speed so found can be ascribed to variations in the form of the stimulus alone, free from the complication of the intensity effect. This method, it is to be remarked, is not subject to any restriction as to wave-length or intensity, but may, with equal assurance, be extended to any such conditions as desired.

Practically, this condition was met by using for the bright field a milk-flashed glass transilluminated by a 60-watt tungsten-filament frosted lamp, both mounted on a calibrated photometric track. The lamp was operated at 110 volts (5 volts below its rated voltage) from a storage battery, and controlled by rheostat and voltmeter. The rotating sectored disk was placed as close as possible to the milk-glass, and a 12-diopter trial-case lens 158 mm from the disk formed an image of the disk on the pupil of the observing eye, 179 mm from the lens. Immediately in front of the lens was placed a screen of cardboard, in which was punched a hole 3/8 inch (9.53 mm) in diameter. To the left of the hole, and at the same distance (3/8 inch) from its center, was a small fixation mark. Close to the eye was an eyepiece, containing a 6-diopter lens, so located that the cardboard screen was at its principal focus. Consequently the field, seen through this lens, and the fixation mark at its left, were clearly visible without accommodation. With this arrangement, properly adjusted, the field was seen to light up or darken uniformly and simultaneously at all points as the disk was slowly moved by hand, by reason of the fact that the radial border of the opening of the disk was imaged in the plane of the pupil of the eye.

A standard average brightness of the field as seen by the eye was set at 25 millilamberts. Three different conditions were used. First, the field at 25 ml was viewed in the dark, a black card was used for the field-opening in front of the 12-diopter lens, and the card was not illuminated. Second, a white card was used illuminated, by a second lamp with controlled voltage mounted in a small box to the right of the eyepiece, to equal brilliance with the field at 25 ml. Third, the same, except that the average brightness of the field was reduced to 5 ml or one-fifth of the standard.

For brevity, these three conditions will be referred to as 25–0, 25–25 and 5–25, respectively, the two numbers in each case designating in order the average brightness of the field, according to the Talbot-Plateau law, and the steady brightness of the ground or surroundings in which it is seen.

Six ratios of open phase to cycle-time (φ) were used: 0.0673, 0.20, 0.50, 0.80, 0.9674 and 0.9824. The corresponding disks were cut from black cardboard, the first four with five openings, giving five cycles per revolution, the last two with two. Of these last, the obscuring sectors were naturally quite narrow. The first disk ($\varphi = 0.0673$) was found to give an unduly high speed at the critical point, when preliminary results were examined. This was traced to an unequal spacing and width of the five openings, which superimposed a slow cycle upon the one designed. The disk was therefore rebuilt by first enlarging the openings somewhat, carefully revising the spacing of them, and attaching small jaws of the same cardboard to act as the radial edges of the openings, using a carefully cut and carefully placed sector as a guide, against which the jaws could be slid as they were glued in place.

Daylight was excluded from the experimental room while the readings were being taken, and the regular lighting was always on. This consisted of a single fixture, central in the room, located in a vertical almost abreast of the subject about 5 feet to his left, and at a height of 10 feet from the floor. It yielded an average illumination of 3.6 footcandles at table height (36 inches) in the working portions of the room. The walls of the room were of a medium gray and there was no "glare."

The subject was asked to vary the speed of the disk until it was, in his judgment, just at the point of disappearance of flicker; the speed was

then read from a Weston tachometer. Ten readings were thus taken in succession, the apparatus was reset for another group of conditions and ten more taken, and so on. It happened not infrequently that the first few readings came out unduly high or irregular, as compared with the whole ten. In such cases, the readings were continued until the early and irregular ones could be omitted from the calculation, and ten reasonably consistent ones remained.

The results obtained by the writer (subject *C*) are plotted in Fig. 2, and those from the two other subjects, *B* and *S*, in Fig. 3. Each point represents 40 settings (four groups of 10), except in the case of *S*, for which the number is 80, and except at one point (25–0, $\varphi = 0.9824$) for *C*, where the number is also 80. Probable errors of the final averages, computed from the averages of 10 are: for *C*, 0.13 to 0.36; for *B*, 0.70 to 1.45; and for *S*, 0.30 to 1.20 cycles per second.

Occasionally it would happen that inordinately

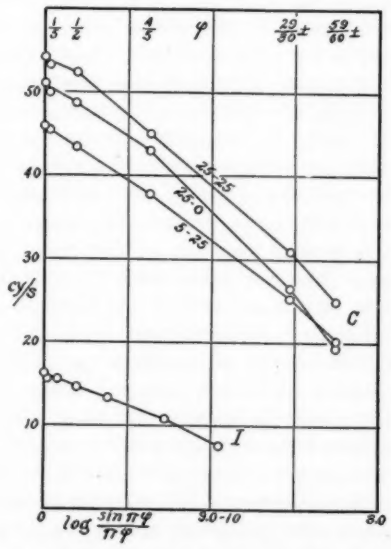

FIG. 2. Critical frequency plotted against the logarithm of the relative amplitude of the first periodic component of the Fourier equation for the stimulus cycle $\log [(\sin \pi\varphi)/\pi\varphi]$, for various ratios φ of light phase to period. Upper curves, results of the present work, intensity factor excluded, for subject *C*: 25–25, average brightness of field 25 ml, brightness of ground 25; 25–0, average field 25 ml, ground dark; 5–25, average field 5 ml, ground 25. The short curve below represents the results of Ives shown in Fig. 1, replotted to this coordinate system.

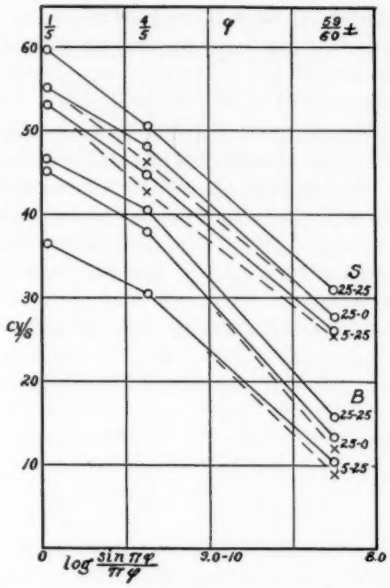

FIG. 3. Results from subjects *S* and *B* of the present work. Explanation same as for *C*, Fig. 2.

high settings would be made and maintained with passable consistency throughout the whole group of 10 settings. For this reason the groups of 4 or 8 averages were examined by Chauvenet's criterion, and the aberrant average removed if it greatly exceeded the limiting deviation. Eliminations of one average were thus made in 4 cases, and elimination of two (out of 8) in 1 case; and no eliminations were indicated by this test in the results of *C*. In Fig. 3 the circles and solid lines represent the averages of all results, and the crosses and broken lines indicate the result after these eliminations. The cause of these aberrantly high critical speeds was not ascertained.

The manner of plotting the data in Figs. 2 and 3 needs explanation. In his discussion, Ives (pp. 265, 268, 352)[2] assumes that the frequency at which flicker disappears is dependent upon the first periodic term in the Fourier series which represents the stimulus intensity as a function of time. For the case of the abruptly changing or square topped stimulus, in which the ratio φ of the light period to cycle time is variable, that series gives, for the intensity at any time t:

$$I_t = I_{av}[1+2([(\sin \pi\varphi)/\pi\varphi] \cos \omega t + [(\sin 2\pi\varphi)/2\pi\varphi] \cos 2\omega t + [(\sin 3\pi\varphi)/3\pi\varphi] \cos 3\omega t \cdots)]. \quad (1)$$

This expression is different in form only from that given by Ives (Eq. (4), p. 265),[2] in that the amplitude has been placed at 0.5, its maximum, and the expression is cast in terms of the average intensity I_{av}, equal to $I\varphi$, since these values are constant for all results in any one curve in the present figures.

From this, according to the reasoning previously used, Eqs. (9), (10),[1] the first photochemical effect at any time t (stage II) will be:

$$x_t = x_{av} + 2I_{av}(C/\omega)([(\sin \pi\varphi)/\pi\varphi] \sin \omega t + \tfrac{1}{2}[(\sin 2\pi\varphi)/2\pi\varphi] \sin 2\omega t + \tfrac{1}{3}[(\sin 3\pi\varphi)/3\pi\varphi] \sin 3\omega t \cdots). \quad (2)$$

It will be seen from this that insofar as they depend upon φ the amplitudes due to the first fluctuating term of this series will be represented relatively by $(\sin \pi\varphi)/\pi\varphi$, and since the results discussed in an earlier paper indicate that the critical speed varies, quite accurately over a considerable range, as the logarithm of the amplitude of the first periodic term, the speeds are here plotted against the logarithm of $(\sin \pi\varphi)/\pi\varphi$, for the purpose of showing whether the results agree with that rule for various values of the ratio φ.

The results of Ives are here shown in Fig. 2, replotted in this way for comparison. They lie much lower than the curves from the present results, no doubt owing to the fact that he worked at a very much lower intensity, within the range of scotopic vision. In this connection it must also be remembered that he used a light of selected spectral blue, instead of the integral light from an incandescent filament. But principally, attention is to be called to the fact that his results as plotted in Fig. 2 fall very close to a straight-line relation with $\log [(\sin \pi\varphi)/\pi\varphi]$. In view of what is to follow this fact should not, however, be stated as a law.

For all three subjects of the present work the highest critical frequencies were obtained under the 25–25 condition (field and ground both of standard brightness). Arrangement 25–0 came next (standard field seen in the midst of darkness), while the reduced field seen on the standard ground (5–25) was lowest of all. This result is a parallel to that obtained by the writer[3] some years ago for simple brightness-difference perception, investigated with reference to the brightness of ground upon which the test field was seen. Differential sensitivity, measured by the inverse of the Weber fraction $\Delta I/I$, was highest when the test field was set in surroundings of about its own brightness; sensitivity was somewhat lower when the surroundings were darkened; and much lower, eventually to an extreme degree, as the surroundings were increased in brightness over that of the test field.

Of the nine curves, from the three subjects *C*, *S* and *B*, shown in Figs. 2 and 3, it is to be said that those of *S* indicate, in general, the highest speeds, and fail to show any definite curvature. In this case then, the relation between speed and the logarithm of the amplitude is a nearly linear one. This subject's results were however rather erratic in spite of the greater number entering into the averages, as will be seen by noting the change due to the elimination of the aberrant values, indicated by the broken lines. The results of *B*, on the contrary, are the lowest of the three, and at the same time show the greatest curvature. Finally, the results of *C* are intermediate between those of the others in both respects. Since they are also more complete, and have the smallest probable errors of the three sets, they are selected to use as the basis of discussion.

It will be seen from Fig. 2 that these curves are longer than that given by Ives' results. The reason is that they extend to a much larger value of φ. Ives stopped with $\varphi = 11/12$, while the present results go as far as $\varphi = 0.9824$ or about 59/60. Although the curvature is not great, it would be difficult to agree that the plotted data represent straight-line relations. In parts the curves are straight, but it would be impossible to point out any striking or uniform tendency in that particular among the three.

The results of *C* show also a feature which does not appear in the others. That is, the approach and crossing of the 25–0 and 5–25 curves as φ comes near to unity. This was a surprise for which no explanation is at hand.

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[3] P. W. Cobb, *The Effect on Foveal Vision of Bright Surroundings—IV*, J. Exper. Psych. **6**, 540–566 (1916).

The relation between the 25–25 and the 5–25 curves represents the intensity relation, as it works out with constant bright surroundings. The logarithmic relation is here, however, not fulfilled unless we assign different coefficients for different values of φ. The intensity of the intermittent field in the upper of these is five times that of the lower; the difference between the logarithms is therefore constant for all values of φ, but the difference of frequency in cycles per second is not constant, varying from a maximum of 9, to a minimum of 5 at the point where φ is greatest. This feature is quantitatively about the same in the case of *B*, while *S* conforms much more closely to the logarithmic law of intensity. However, this is somewhat aside from the main purpose of the present discussion.

In the previous paper[1] it was pointed out that Ives' assumption, that the first periodic term in the Fourier series representing the time equation of the stimulus determines the fusion point, failed to apply in at least one case in his own results. This was shown by plotting together (a) the results obtained by him with a stimulus of simple sinusoidal form of variation, against the amplitude measured as its maximum deviation from the mean value and (b) the results from a square-topped or abruptly changing stimulus against its amplitude measured, not in terms of the actual width of stimulus variation, but as the maximum deviation of the first periodic term in the Fourier series representing the stimulus cycle. The two curves, when the latter was thus plotted, came very near to coincidence at higher amplitudes, which would tend to show that here it is indeed the first component which determines the end point of flicker. This, however only gives the more point to something quite different which happens at low amplitudes.

The sinusoidal stimulus characteristically shows something which the other does not show at all, that is, two flicker points. One of these is a point at which flicker begins with increasing speed; the other is the point at which it disappears. The first rises and the second falls (in speed) as the amplitude is reduced, until at an amplitude of 1/16 (the maximum possible being 0.5) the sinusoidal stimulus could not be made to yield flicker at any speed whatever (Ives,[2] p. 262). Yet the square-topped stimulus continues to show flicker at this point. From this, it is impossible to escape the conclusion that the first periodic component of the square-topped stimulus *alone* is ineffective in that case, since the sinusoidal stimulus which yields the exact equal of that component by itself absolutely refuses to flicker. It follows that such flicker as there is must be due, in part at least, to one or more other components of higher frequency.

It may be proposed then, that in certain cases it is not the fundamental alone but the combined and more complex wave due to the summation of it and one or more of the components of shorter period, which determines the end point of flicker. Indeed, if we hold to the feasibility of resolving the stimulus into periodic components at all, it would seem that there is no alternative. If the first periodic component is of too low amplitude to flicker at any speed, how could flicker be produced at any speed whatever, for example, by the second or third alone, which are of no greater amplitude? (Compare Fig. 4.) In

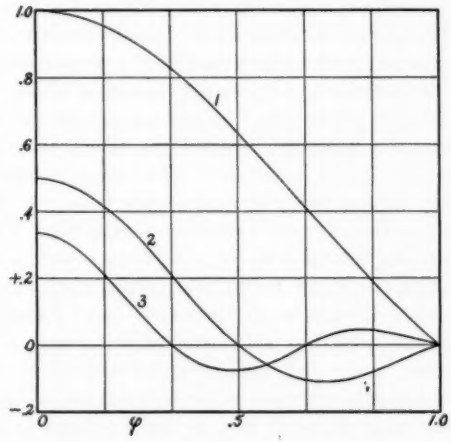

FIG. 4. Curves 1, 2 and 3, the relative coefficients of the first, second and third periodic terms of the cycle of change in the state of the hypothetical photochemical cell, Eq. (2), $(\sin \pi\varphi)/\pi\varphi$, $(\sin 2\pi\varphi)/4\pi\varphi$ and $(\sin 3\pi\varphi)/9\pi\varphi$, representing relative amplitudes, plotted against the ratio φ of light phase to period.

such a case then, in which it can be empirically shown that the first is inadequate, it must be that two or more of the periodic components conspire to bring about the flicker effect. If that is true, one wonders what is gained for the

subject by resolving the stimulus into periodic components in the first place.

It will be remembered from the previous paper, Eqs. (18) to (21),[1] that the condition under which a sine-wave stimulus will cease to flicker by reason of reduced speed is one which does not admit of the low order of approximation used in the transition from stimulus pattern to photochemical effect (stage I to stage II) by Ives and followed here in deriving Eq. (2) from (1) above, by simple integration. That transition, as was pointed out, is approximately valid only when the lag may be assumed to be very large between the stimulus change and the resulting change in level of the photochemical condition of the cell; whereas the corresponding transition at the point where flicker from a sinusoidal stimulus *ceases with decreasing speed* presupposes a small lag and necessitates the introduction into the coefficient of the corresponding term at stage II of a factor which, in this paper, is now mentioned for the first time, and which radically alters the relation from that given in the transition between (1) and (2) above.

This serves to emphasize a point made in the last paper, to the effect that in the present state of our knowledge, an experimental analysis of flicker will proceed to the best advantage by the exhaustive study of a simple sinusoidally varying stimulus, before going on to the more complex forms whose simplicity consists only in the comparative ease of technical production. The sinusoidal stimulus is the only form which goes through the three theoretical stages of the Ives theory with change in amplitude (and possibly in phase) only, but without change in form; and further, since the experimental technique of summating two sinusoidal stimuli having periods in the ratio of one to two, or one to three, is by no means insurmountable, various combinations of amplitude and phase could be provided, for the purpose of finding out how these factors supplement one another in determining the critical frequency. Such a study would at least yield evidence bearing on the value of Fourier analysis in the study of flicker.

In Fig. 4 are shown the relative amplitudes of the first, second and third periodic terms in Eq. (2) above, as their values vary with progressive increase in φ from zero to unity, while the average intensity $I\varphi$ is kept constant. The general decrease in amplitude, because of the presence of φ in the denominator, is reflected in the general decline in critical frequency shown by the results.

As far as it is permissible to apply here the numerical result obtained above from Ives' data, the point at which the first periodic component alone becomes incapable of producing flicker at all would be where its amplitude becomes less than, say 0.08. From Fig. 4 we find this value about at the point where φ has the value 11/12 or 0.917, and from here on to unity we may suppose that the fundamental is not alone the determinant of the flicker point. This region includes the last two plotted points of the present work.

The bend in the curves is not easy to account for. It may be that we are not yet compelled to account for it. The consideration mentioned above does not mean that the critical speed at low amplitudes would necessarily deviate from the logarithmic (or any other) relation to any great extent. The effectiveness of the second and third components, supposing that they are effective, should increase as φ departs from the value 0.5, since their amplitudes undergo aggregate increase in either direction, relative to that of the fundamental (compare Fig. 4). This, however, would result in curves with their convexity downward, rather than upward, if we are thinking of deviations from a simple logarithmic law. It is further possible that the critical-frequency-amplitude relation is not essentially a logarithmic one, but that like many other simply-stated biological relations, the logarithmic law is a statement which is more or less accurate under certain limited conditions only.

Summary

(1) The effect of the ratio of light phase to period on the critical frequency of flicker is investigated by a method in which the effect of intensity is experimentally eliminated by equalizing the average intensity of light admitted to the eye according to the Talbot-Plateau law.

(2) With a standard intermittent field of 25 millilamberts average brightness, the critical frequency was lower when the field was seen in the dark than when it was set in steady and

uniform surroundings of 25 ml brightness. An intermittent field of 5 ml average brightness, seen in surroundings of 25 ml gave critical frequencies considerably lower than either of the above.

(3) These findings run parallel to the relation between sensitivity to brightness difference and comparative brightness conditions of test field and ground.

(4) The effect of varying the open or bright fraction of the cycle is investigated with respect to the amplitude of the first periodic term of the Fourier series which represents the stimulus cycle; and critical frequency was found to deviate from a linear relation to the logarithm of this amplitude, in the sense that it was relatively high when the light fraction of the cycle time was equal to one-half, and relatively low as the fraction approached zero or unity.

(5) The curvature of the plotted results is not great, the general tendency for the critical frequency to decrease as the relative duration of the light phase is increased is strong, and there are no reversals.

(6) The curvature is the opposite of that required to explain the results by the hypothesis that the second, third, etc., periodic terms of the series jointly contribute to the flicker effect since, with respect to amplitude, these are quantitatively of least importance when the light phase is one-half the period, and increase in relative importance as the ratio of light phase deviates from this toward zero or toward unity.

(7) Certain difficulties are discussed, which stand in the way of explaining the physiology of flicker by means of the resolution of the stimulus cycle into sinusoidal components.

Resistance and Radiation of Tungsten as a Function of Temperature

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IN a paper[1] by one of the present authors and Dr. Worthing, data on the radiating characteristics of tungsten obtained as a result of study that extended over a period of about ten years prior to 1922 were given. Some data published[2] since, indicated that there were some discrepancies in the temperature-resistance data given in that report. To check this, the Lamp Department Wire Division furnished some very pure 10-mil tungsten wire that had been very carefully drawn through tested dies so as to produce wire that was as uniform and as nearly round as possible. Mr. P. P. Tarasov, metallurgist at the Wire Division, made some tests of this wire both before and after it was used in our experiment and has the following to say concerning it:

> Structurally this represents a typical non-sag tungsten wire. As viewed on a microscope, its structure consists of very long grains occupying, in places, the whole transverse section of the wire. Occasional small grains are found "imbedded" within the large grains or at grain boundaries.
>
> At high magnifications the metal appears quite homogeneous and free from inclusions, so that, if it contains any impurities or admixtures, such as carbon, iron, silicon, potassium, etc., they must be present there in minute quantities and in a state of extreme dispersion or in solid solution with tungsten.
>
> The surface of the wire bears many grooves which apparently are "drawing marks." A grain boundary with few small grains can also be observed on these photomicrographs.
>
> The measurement of the diameter of the wire with a micrometer checked by measurements of photomicrographs of a cross section showed that the wire is slightly (less than two percent) out-of-round.

The temperatures given in this paper are on the scale that has been used at the Nela Park Laboratories for the past ten years. On this scale the gold point is 1336°K and C_2 is taken 14,330μ°. Previous work[3] shows that temperature measurements made at Nela Park agree well with like measurements made at the various Standardization Laboratories. Worthing's average values[1] for the emissivity of tungsten were used to reduce the brightness temperature obtained with the optical pyrometer to true temperatures. While the wire used in this experiment had evidences of die marks it had a very good polish which was probably quite similar to that of the filaments used in previous experiments.

Several lamps with long hairpin filaments having potential leads, were made of samples of this wire and the current and voltage measured for temperatures between 1200 and 2800°K. The logarithm of the resistance was plotted against the logarithm of the temperature and well within the limit of the measurements, the relation was found to be linear with a slope of 1.195 which agrees very well with the average of the previous result[1] of 1.20 for the slope of this logarithmic relation. It might be added here that no indication of anything except a linear relation for the logarithm of the resistance plotted against the logarithm of the temperature for a range between 1200 and 3000°K has ever been found in this laboratory. This statement is based on measurements of very many lamps and very many samples of tungsten wire. Since one part of this test was to study the characteristics of tungsten for temperatures between 1200°K and room temperature it was necessary either to eliminate the effect of losses[4] due to the heat conducted away by the leads or in some manner to make corrections for the effect of these losses.

Calculations similar to those given in a paper by Langmuir[5] and associates show that for a

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[1] W. E. Forsythe and A. G. Worthing, Astrophys. J. **61**, 146 (1925).

[2] Zwikker, Physica **5**, 249 (1925); Jones, Phys. Rev. **28**, 202 (1926); Geiss, Ann. d. Physik **79**, 85 (1926).

[3] W. E. Forsythe, Phys. Rev. **38**, 1247 (1931).

[4] Langmuir, MacLane and Blodgett, Phys. Rev. **35**, 478 (1930); E. P. Hyde, F. E. Cady, A. G. Worthing, Trans. Illum. Eng. Soc. **6**, 238 (1911).

[5] Langmuir, reference 4.

10-mil filament (0.025 cm in diameter) of tungsten wire an appreciable change of temperature due to the end conduction does not extend further than about 40 cm for a temperature of 450°K. Accordingly, to extend the test to very low temperatures a lamp was made up of this selected wire with a hairpin filament with an overall length of the filament of about 110 cm with potential leads attached across the lower 25 cm of the filament. All of the metal inside of the lamp that constituted the potential leads was tungsten and all joints were carefully welded. The part of the potential leads attached to the side of the large filament were coiled filaments made of wire about 0.02 mm in diameter. The amount of heat conducted away by wire of this size is very small. This lamp was first made up gas-filled and aged for about two hours at 3000°K. Then in evacuating the bulb the pumps were kept running several hours with the filament heated to about 1000°K and the lamp heated in a furnace to about 350°C. Attached to the side of this lamp was a tube containing charcoal. During the evacuation of the lamp this charcoal was heated by a specially attached furnace to about 400°C for about an hour. First, the temperature, current and voltage of this lamp were measured for a range of temperatures from 1100°K to 2800°K. This was repeated four times, half of the readings being taken when the temperature was increased and the other half when the temperature was decreased, the filament being allowed to operate at each current for fifteen minutes before readings were made. The voltage was then measured for a range of currents that corresponded to temperatures from 1200°K down to 450°K. Readings on the current and voltage from 600°K down to room temperature were made with the lamp mounted in the open room and also mounted in an oil bath which was surrounded by an ice bath. When the lamp was mounted in the oil bath about 30 cm of the bulb projected above the oil and about 50 cm was immersed in the oil. The results under these two conditions of operation for this temperature range agreed very well. Calculations show that at 660°K there should be about 0.5 percent difference in the input whether the lamp was surrounded by an ice bath or at room temperature, which in this case was 20°C. The experimental results agreed with this calculation. The resistance as measured with the lamp in the ice bath came out too high in comparison with the room temperature values which was disturbing until this was recognized as a case of reverse end loss, that is, since the base of the lamp was not inserted in the ice bath, heat was being conducted into the filament from the base, which was at room temperature, thus heating the filament slightly and giving too high value for the resistance. When measurements were being made at these low temperatures liquid air was kept on the side tube containing the charcoal so as to keep the vacuum as low as possible, since the assumption is made that there are no losses from the lower end of this filament except the radiation losses. To see whether the end losses affected the characteristics of the central 25 cm of this filament at low temperatures an attempt was made to eliminate entirely these end losses. While most of the lamp was inserted in the ice bath the voltage and current were measured with the base of the lamp surrounded by a furnace which was so operated that the base was at approximately the same temperature as the lower end of the filament. With the base of the lamp and the lower part of the filament at the same temperature, there should be no conduction of heat either to or from the lower part of the filament. By this method readings were made for a temperature range between about 470 and 319°K. This experiment showed that the calculations based on Dr. Langmuir's equations were correct in that 40 cm of tungsten 0.02 cm in diameter as used for this filament was sufficient to eliminate end losses for a temperature of 450°K.

After all these measurements were made on this lamp it was broken open and dimensions of the filament determined. The total amount of filament between the two potential leads was very carefully weighed and then its length determined by measuring the length of its trace on a piece of blue-print paper with a measuring wheel. This gave a first determination of the diameter of the wire, with 19.31 for the density of tungsten. A straight piece of this same filament was very carefully weighed and its length directly measured thus permitting a check on the diameter or the length previously obtained. This

method of determining the diameter gives an average that partially corrects for the slight out-of-round found for the wire. To check these values another lamp was made with samples of the same wire, the difference in this case being that the bulb contained two filaments of different lengths. Except that this lamp had no attached charcoal tube, it was treated in every respect like the first, and measurements were made for a temperature range of 1500 to 2800°K. The final results agreed with the average of those from the first lamp to a little better than one-half percent in specific resistance.

Another test on the temperature-resistance relation of tungsten wire was made with some very pure 25-mil (0.064 cm in diameter) tungsten wire that was manufactured in 1926. This filament was about 60 cm in length with potential leads across the lower 25 cm. It was first made up in an atmosphere of nitrogen-argon mixture and aged for more than an hour at 3000°K and then evacuated and the test made on the filament in a vacuum. The resistance was measured for three temperatures between 1600 and 2200°K after which the lamp was broken open and the dimensions of the wire measured. These values check the results on the first lamp to about 0.2 percent in resistance.

Since all three of these tests included a determination of the physical dimensions of the filament along with the temperature and resistance, the checks are thought to be very satisfactory particularly since the results on these three lamps were checked relatively at high temperatures by measurements of the temperature and electrical characteristics of a number of lamps made of the same wire.

The logarithm of the specific resistance was plotted against the logarithm of the temperature for the entire temperature range covered by these tests and the curve was found to be linear from 1200°K to 2800°K with a slope of 1.195, with only a slight change in the slope somewhere between 1200°K and room temperature. A straight line drawn through the log temperature-log resistance for 1200°K and 293°K as experimentally determined has a slope of 1.209. The slope probably does not change abruptly at a temperature of about 1200°K but sufficient data were not available to determine the shape of the curve at the low temperature region.

Dr. Worthing, while working in this Laboratory, calibrated a number of regular 40-watt tungsten lamps for the temperature-resistance relation for a temperature range from room temperature up to about 400°C by measuring the filament resistance with the lamps in a furnace held at definite temperatures which were measured with a calibrated thermocouple. The results on ten lamps for this range agree very well and give an average slope of 1.20 for the log temperature-log resistance relation.

While the slope of the log temperature-log resistance relation agrees with the previous results[1] the actual values of the resistance for a definite temperature are not in agreement, being about six percent lower and agreeing much better with like values of Jones[2] and of Geiss.[2] We are at a loss to account for the difference between the values of specific resistance as found for these filaments at this time as compared to the previously published results. Part of the change might be due to an actual change in the specific resistance of the tungsten wire as it is manufactured. The ratio of the resistance of a straight filament lamp at room temperature to the resistance of the lamp at its operating condition had been measured for very many lamps previous to the publication of the former results. One reason for a belief in the accuracy of these previous results was that they were in very exact agreement with the ratio of hot to cold resistance of these various tungsten lamps when this ratio was corrected for the effects of end conduction. It is to be noted that the value obtained for the specific resistance of tungsten at room temperature agrees with the value given by Jones[2] to about 0.2 percent.

The data and calculations for the previously[1] published results have been gone over and no error can be located. The agreement of the results on the total emissivity obtained in this test with previous results make it all the more difficult to understand this difference, unless there has been a change in the specific resistance of tungsten wire at high temperatures, since the specific resistance-temperature relation and the total emissivity-temperature relation depend

TABLE I. *Characteristics of tungsten wire.*[1]

Temperature (degrees K) T	Resistivity (microohms cm) ρ[2]	$\frac{T}{\rho}\frac{d\rho}{dT}$	Normal brightness (candles per cm²) B_n	$\frac{T}{B}\frac{dB}{dT}$	Total emissivity e_t	Total radiation intensity (watts/cm²) n[3]	$\frac{T}{n}\frac{dn}{dT}$
273	5.05	1.209			0.022	0.00070	
293	5.50	1.209			.023	.00095	
300	5.65	1.209			.024	.00110	
400	8.00	1.209			.034	.00495	
500	10.48	1.209			.042	.0150	
600	13.07	1.209			.052	.0385	
700	15.75	1.209			.062	.0850	
800	18.51	1.209			.074	.1730	5.47
900	12.35	1.209			.089	.333	5.50
1000	24.26	1.209			.105	.600	5.51
1100	27.23	1.209			.121	1.01	5.53
1200	30.26	1.195			.138	1.63	5.53
1300	33.29	1.195			.156	2.54	5.52
1400	36.37	1.195			.174	3.82	5.48
1500	39.50	1.195			.192	5.54	5.41
1600	42.67	1.195	0.94	15.2	.207	7.74	5.31
1700	45.88	1.195	2.30	14.4	.222	10.58	5.18
1800	49.12	1.195	5.15	13.7	.236	14.15	5.02
1900	52.40	1.195	10.40	13.0	.248	18.45	4.89
2000	55.71	1.195	20.00	12.3	.259	23.65	4.80
2100	59.05	1.195	35.9	11.7	.269	29.85	4.73
2200	62.42	1.195	61.0	11.2	.278	37.20	4.68
2300	65.82	1.195	100.1	10.8	.286	45.70	4.64
2400	69.25	1.195	156.0	10.3	.294	55.70	4.61
2500	72.71	1.195	234.0	9.9	.301	67.2	4.59
2600	76.20	1.195	345.	9.6	.309	80.6	4.57
2700	79.71	1.195	495.	9.2	.315	95.6	4.56
2800	83.25	1.195	690.	8.9	.321	112.5	4.55
2900	86.81	1.195	950.	8.6	.329	132.5	4.54
3000	90.40	1.195	1270.	8.3	.334	154.5	4.53

[1] Data for well aged, very pure tungsten wire.
[2] These values depend on the dimensions at room temperature.
[3] Surrounded by a black body at 0°K.

upon the same measurement of current, voltage and physical dimensions of the filament.

By using the data from which the specific resistance-temperature relation was determined, a redetermination was made of the total emissivity of tungsten for the temperature range between 2800°K and room temperature. The temperature of the filament was measured with an optical pyrometer for temperatures from 2800°K down to about 1200°K and temperatures between this and room temperatures were read from a straight line log temperature-log resistance plot between 1200°K and room temperature. As has already been pointed out a straight line very nearly represented the log temperatures. It was only necessary to increase the slope from 1.195 to 1.209 at about 1200°K in order to have it pass through the room temperature resistance point. The values for different temperature of the specific resistance, the total emissivity and some other data on tungsten read from a smooth curve drawn as nearly as possible through the different experimental points are shown in Table I. The values of the total emissivity of tungsten given in Table I agree with the previously published values to better than 0.5 percent of their values over the range from 1200°K to 2800°K. By using known values of the spectral emissivity[6] for the visible and infrared spectrum, the total emissivity was calculated for 1000°, 600°, 400° and 273°K for the filament surrounded by a black body at 0°K. This calculation shows that for a temperature of 600°K the total emission would be changed by only about four percent whether the surroundings were at 0°K

[6] A. G. Worthing, Phys. Rev. **10**, 377 (1917); W. Weniger and A. H. Pfund, ibid. **14**, 427 (1919).

or at 273°K and that the difference would be less than one percent whether it were surrounded by a black body at 293°K or at 273°K. These calculated values agreed with the experimental results to within the accuracy of measurement.

It is difficult to estimate the accuracy of true temperature measurements of a metallic surface. In the case of tungsten the heat treatment above described has always been considered to give the tungsten wire a good polish of a definite nature. This is borne out by the agreement of our temperature resistance relations obtained during many years. If the higher temperatures were corrected so that a straight line log-temperature log-resistance curve would pass through all the points from room temperature to higher temperature the high temperature would have to be corrected by about 25°C. It is not thought that any such error exists in the true temperature scale of properly aged tungsten filaments.

The brightness of tungsten is related to that of a black body by the following relation:

$$\log B_w = \log B_{bb} + \frac{C_2 \log e}{\lambda}\left(\frac{1}{T_c} - \frac{1}{S}\right).$$

B_w is the brightness of tungsten at a color temperature of T_c corresponding to a brightness temperature of S for wave-length λ; B_{bb}, the brightness of a black body at a temperature of T_c. The part of the equation at the right of the plus sign is the logarithm of what has been called the color emissivity of tungsten. This equation would give the exact brightness of tungsten if the color match meant an exact spectral match but since it has been shown[7] that this is not the case a slight correction has to be made for this difference. The correction due to a color match not giving the same spectral distribution for tungsten is about 0.7 percent at 2600°K and about 0.3 percent at 1600°K. With this equation together with these corrections and the brightness of a black body as published by Ives[8] the brightness of tungsten has been calculated and is shown in Table I. These values agree to better than a percent with the previous results which were the average of brightness obtained by this method and by two other methods involving direct measurements.

[7] W. E. Forsythe, J. O. S. A. and R. S. I. **7**, 1115 (1923).
[8] Herbert E. Ives, J. O. S. A. and R. S. I. **12**, 75 (1926).

Journal of the Optical Society of America
The Review of Scientific Instruments

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS*

FOR THE PERIOD FROM JANUARY 1, 1933 TO DECEMBER 31, 1933

Cash on Hand, January 1, 1933:		
Checking Account	$ 107.52	
Interest Accounts	2,294.26	$2,401.78
Receipts:		
In account with Optical Society of America:		
Sale of back numbers (Volumes 1–5)	$ 37.40	
Advertising collected for O.S.A.	71.00	
Payment from Treasurer (for 1932)	1,122.00	$1,230.40
In account with Scientific Apparatus Makers of America:		
Advertising		$ 858.50
Miscellaneous receipts:		
Sale of back numbers (Volumes 6–)	$ 71.48	
Dartmouth College—Special subvention	600.00	
Reimbursement	.10	
American Institute of Physics, Special cash fund	391.86	
Interest on interest accounts	71.82	$1,135.26
Total		$5,625.94
Disbursements:		
Printer	$1,834.12	
Office expenses	61.43	
American Institute of Physics	401.50	
Miscellaneous	39.93	
American Institute of Physics, Payments against Special cash fund	306.70	$2,643.68
Cash on Hand, December 31, 1933:		
Checking Account	$ 616.18	
Interest Accounts	2,366.08	$2,982.26
Total		$5,625.94

The above is a correct statement of accounts to the best of my knowledge and belief.

(*Signed*) F. K. RICHTMYER, *Editor.*

We, the undersigned, duly appointed to audit the accounts of the *Journal of the Optical Society of America* and *The Review of Scientific Instruments* at the close of business, December 31, 1933, hereby certify that we have examined the statement of receipts, and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, therefore, certify that the above statement is a true statement of the accounts of the *Journal of the Optical Society of America* and *The Review of Scientific Instruments* as of December 31, 1933.

(*Signed*) G. E. GRANTHAM
F. R. HIRSH, JR.

* Cash transactions occurring in 1933 mainly as a result of business done in 1932. (Operation of these journals in 1933 was delegated to the American Institute of Physics, Incorporated.)